An Applications-First Approach to Calculus II through Differential Equations Modeling

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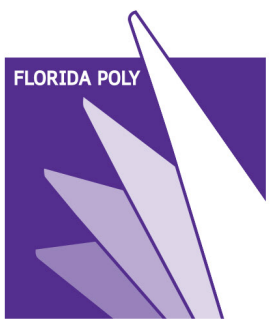

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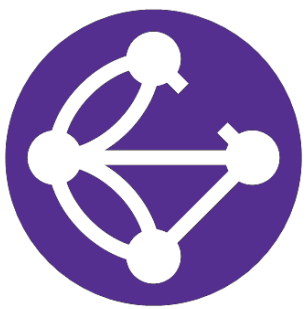


Introduction

Who I Am

- I have been an Applied Mathematics instructor at Florida Polytechnic University for three years
- I am very much interested in mathematical modeling
- I am here today to tell you about some reforms we made to Calculus II starting last Fall

Calculus Reform

- Many of our students at Florida Poly struggle with the Calculus sequence
- Particularly in Fall, most of them arrive fresh from high school
- Many of them are not yet prepared to work through longer and more conceptually challenging problems

Calculus Reform

- We've gradually edited our Calculus sequence in response
- Our most significant changes were made last Fall
- I suggested an applications-first approach featuring prominent use of differential equations models

Outline

- My goal today is to tell you about what we did and to convince you to give it a try
- Outline:
  1. Common problems with Calculus II and motivation for restructuring it in the way we did
  2. How we implemented our differential equations unit last Fall
  3. What we learned from the experience and some preliminary results

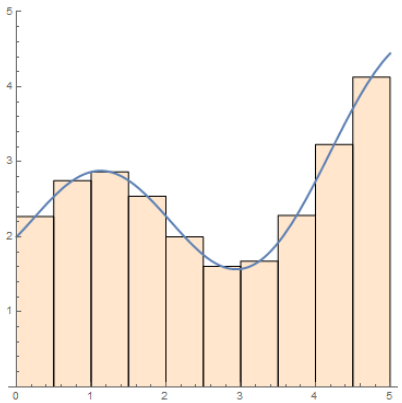
Motivation

Problems with Calculus II

- I believe Calculus II typically serves as a uniquely poor introduction to college-level math
- Common problems include:
  1. The early topics encourage the students to develop unhelpful habits
  2. The introduction is very boring and unmotivating
  3. The topic order lacks structure and obvious end-goals

Problem 1: Unhelpful Habits

- Calculus II typically begins with a review of Calculus I topics
- It is typically followed by integration techniques and applications of integration

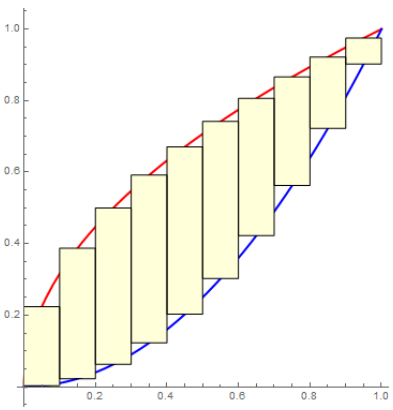


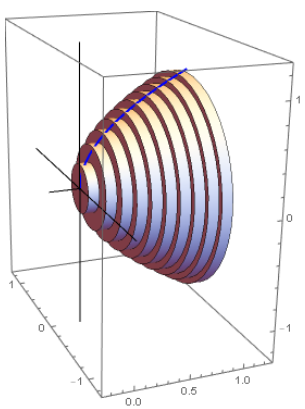
Problem 1: Unhelpful Habits

- Both of these put students into a mindset that the course will be all about memorizing and cranking through formulas
- This really hurts them later in the course
- It's also unrepresentative of real-world STEM work

Problem 2: It's Boring

- The early applications typically discussed in Calculus II are things that no student cares about
- They're very obviously artificial and unmotivating geometric computations
- This makes for a terrible first impression of college-level math





Problem 3: Lack of Structure

- Calculus II is generally taught as a sequence of disconnected advanced topics that could be presented in almost any order
- Major topics typically include:
  - Review of integral calculus
  - Applications of integration
  - Advanced integration techniques
  - Differential equations
  - Sequences and series
  - Power series
  - Parametric and polar coordinates
- There is no overarching end-goal that everything builds towards

An Applications-First Approach

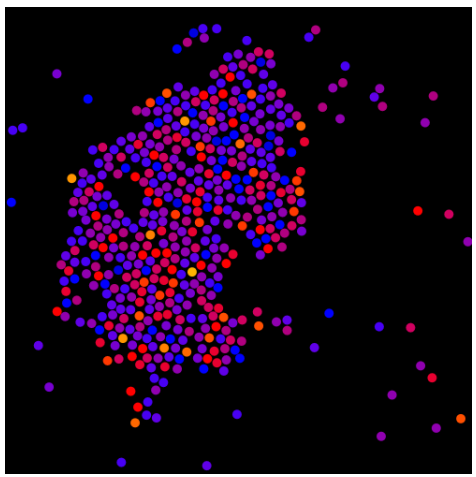
- Our Calculus II redesign was primarily motivated by trying to mitigate these issues
- The general design mantra was “applications first”
  - *Begin* each section by introducing the motivating applications and end-goals
  - Use these end-goals to inform the theory and methods developed within the section

An Applications-First Approach

- The centerpiece of this approach was *differential equations*
  - DEs are the first really interesting, applied topic usually covered in Calculus II
  - There's no reason to hide it in the middle of the course
- There are lots of potential advantages to beginning Calculus II with a differential equations unit

It's Interesting

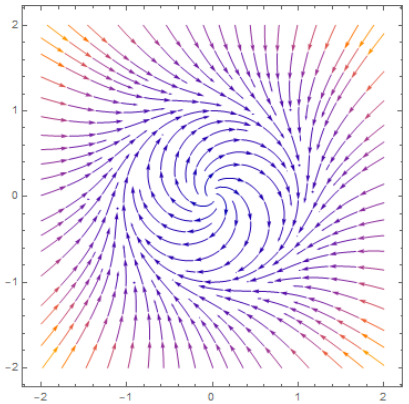
- DEs are much more interesting than most of the typical early Calculus II topics
- They're also very obviously applied and easy to motivate
- DE models allow us to simulate complicated systems that emerge from simple rules



Daniel V. Schroeder, `https://physics.weber.edu/schroeder/md/`

It's Different

- DEs can help to more immediately shake the students out of unhelpful high school habits
- They seem very obviously different than anything covered in Calculus I
- Studying them very obviously requires a different set of skills than what they might be used to from high school-level math



It's Accessible

- There's no reason not to
- DEs can be defined, understood, and analyzed by simply knowing that a derivative represents a rate of change
- As a bonus, Calculus I skills are still embedded within DEs, which allows a DE unit to double as a covert Calculus I review

Implementation

Differential Equations Unit

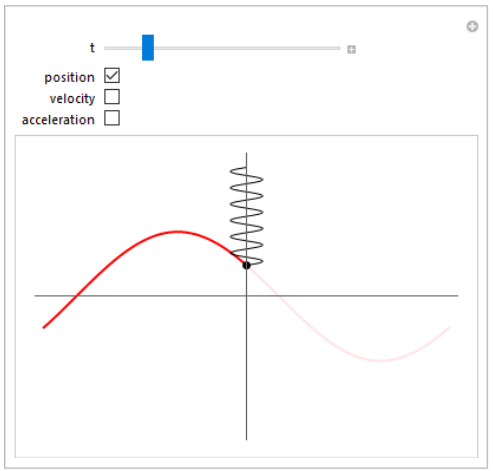
- The stated Course Learning Outcomes for the differential equations unit were:
  1. Determine the behavior of a system given its DE model by:
     - a. solving the DE (exactly or numerically)
     - b. qualitatively analyzing the DE (slope field, phase line, equilibrium analysis, etc.)
  2. Tie a differential equation model to the real-world system it describes by:
     - a. building a model based on a description of what causes a real-world system to change
     - b. interpreting what the terms in a model say about how the real-world system changes

Differential Equations Unit

- The differential equations unit was deliberately designed to *avoid* explicitly reviewing Calculus I material for the first two weeks
- This was to prevent students from immediately getting into a memorization-based mindset

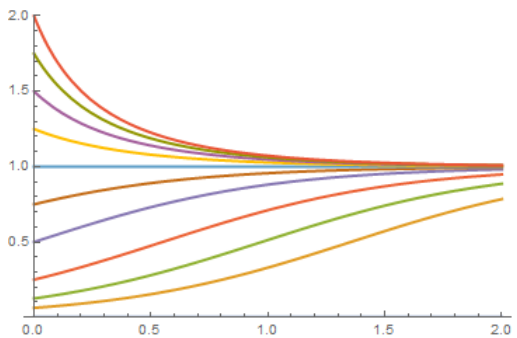
Differential Equations Unit

- The first week consisted of looking at differential equations models from various STEM applications
  - Exponential growth/decay, spring-mass, etc.
- The main goal was to illustrate that differential equations are interesting, important, and widely-used throughout STEM fields



Differential Equations Unit

- Fundamental concepts were discussed during Weeks 1–2
  - Defining a “solution” of a differential equation (this doubled as a covert review of differentiation rules)
  - Slope fields
  - Numerical solution methods
  - Equilibrium analysis
- By the end of Week 2 we could qualitatively analyze or numerically solve a DE



Differential Equations Unit

- Week 3 was dedicated to reviewing the definite integral, the Fundamental Theorem of Calculus, and the Net Change Theorem
- All topics were contextualized in terms of how they apply to differential equations

Differential Equations Unit

- Last Fall we included a couple of differential equations-related projects throughout the semester
- This Spring we made a much more intentional effort to regularly dedicate class time to group work
  - Some projects are meant for the students to learn new material
  - Some are meant as a comprehensive review of a section

Differential Equations Unit

- Projects were made a priority to encourage students to regularly talk about ideas out loud with each other
- This is important for something as conceptually robust as differential equations
- Some topics also make more sense as hands-on projects as opposed to lectures, like Euler's method

Differential Equations Unit

- Week 4 consisted of a comprehensive unit project that involved:
  - defining a model for a population with predation
  - applying equilibrium analysis to determine long-term behavior for different initial conditions
  - numerical solution to simulate the behavior of the population under different circumstances
- This was followed by the unit exam

The Rest of the Course

- The rest of the course is similarly organized into self-contained applications-first units
- A week-long final project is planned to reincorporate material from all units by developing, analyzing, and applying a large, intricate differential equation model to solve a real-world problem

Preliminary Results

Results

- We have only been implementing this approach since last Fall
- It is still a bit too early to make strong conclusions about the effects of these redesigns

Results

- Student performance last Fall was similar to that of previous years' Fall sections
- Exam 1 performance this Spring was much *much* better than in Fall
- The same pattern has occurred for all changes we've made to the course over the past several years

Results

- We always observe a drastic performance improvement from Fall to Spring Calculus II
  - Spring students already have some college experience
  - They've had less time to forget material
  - Fewer of them are products of AP Calculus, and are more ready to apply a broader range of skills

Results

- We've just had our first chance to see the downstream effects of these redesigns in the classes that follow Calculus II
- This semester's Differential Equations instructors are currently teaching many of last semester's Calculus II students
- The students seem to be picking up the qualitative analysis techniques (phase lines and equilibrium analysis) much more easily

Results

- The redesign is certainly not *worse* than the usual version
- There's no reason *not* to use it
- Our instructors preferred the redesign and it's more fun to teach
- The potential benefits are good enough to stay with it

Student Perception

- Sentiment regarding this shift in focus varies a lot from student to student
  - Some appreciate having a strong focus on real-world applications as opposed to just evaluating a lot of integrals
  - However a lot would prefer the class to just be a list of formulas and techniques to memorize

Group Work

- Sentiment regarding group work also varies a lot from student to student
  - There is a long adjustment period
  - Many students are resistant to any format besides lecture
  - However a lot of groups have started working together on homework and studying without being prompted to

Changes in Framing

- Spend less time on the “business” of learning methods for hand computations
- Spend more time messing around with interesting, complex models for fun without needing to exactly solve or fully analyze them

Changes in Framing

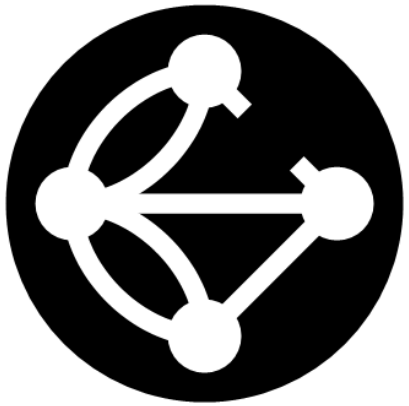
- The weakest area for student performance has consistently been questions about interpreting model terms and parameters
- These questions require no calculus skills: only critical thinking and explaining
- This is something our students need more practice with

Conclusion

Your Input

- The changes we tried are ultimately fairly simple
- There are plenty of potential benefits to improve a student's first college-level math experience
- I would love for you to try this on your own and to hear about the improvements you make

Thank you!



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