Getting to the Good Part: An Applications-First Approach to Calculus II through Differential Equations Modeling

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Introduction

Who I Am

- I have been an Applied Mathematics instructor at Florida Polytechnic University for three years
- I am very much interested in mathematical modeling
- I am here today to tell you about some reforms we made to Calculus II last Fall

Calculus Reform

- Many of our students at Florida Poly struggle with the Calculus sequence
- Particularly in Fall, most of them arrive fresh from high school
- Many of them are not yet prepared to work through longer and more conceptually challenging problems

Calculus Reform

- We've gradually edited our Calculus sequence in response
- Our most significant changes were made last Fall
- I suggested an applications-first approach featuring prominent use of differential equations models

Outline

- My goal today is to tell you about what we did and to convince you to give it a try
- Outline:
  1. Common problems with Calculus II and motivation for restructuring it in the way we did
  2. How we implemented our differential equations unit last Fall
  3. What we learned from the experience and what we will do differently this Spring

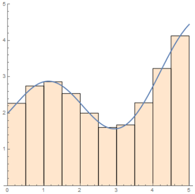
Motivation

Problems with Calculus II

- I believe Calculus II typically serves as a uniquely poor introduction to college-level math
- Common problems include:
  1. The early topics encourage the students to develop unhelpful habits
  2. The introduction is very boring and unmotivating
  3. The topic order lacks structure and obvious end-goals

Problem 1: Unhelpful Habits

- Calculus II typically begins with a review of Calculus I topics
- It is typically followed by integration techniques and applications of integration

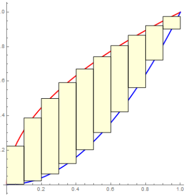


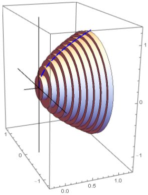
Problem 1: Unhelpful Habits

- Both of these put students into a mindset that the course will be all about memorizing and cranking through formulas
- This really hurts them later in the course

Problem 2: It's Boring

- The early applications typically discussed in Calculus II are things that no student cares about
- They're very obviously artificial and unmotivating geometric computations
- This makes for a terrible first impression of college-level math





Problem 3: Lack of Structure

- Calculus II is generally taught as a sequence of disconnected advanced topics that could be presented in almost any order
- Major topics typically include:
  - Review of integral calculus
  - Applications of integration
  - Advanced integration techniques
  - Differential equations
  - Sequences and series
  - Power series
  - Parametric and polar coordinates
- There is no overarching end-goal that everything builds towards

An Applications-First Approach

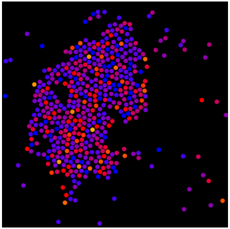
- Our Calculus II redesign was primarily motivated by trying to mitigate these issues
- The general design mantra was “applications first”
  - *Begin* each section by introducing the motivating applications and end-goals
  - Use these end-goals to inform the theory and methods developed within the section

An Applications-First Approach

- The centerpiece of this approach was *differential equations*
    - DEs are the first really interesting, applied topic usually covered in Calculus II
    - There's no reason to hide it in the middle of the course
- There are lots of potential advantages to beginning Calculus II with a differential equations unit

It's Interesting

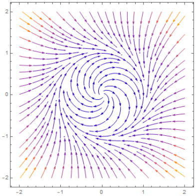
- DEs are much more interesting than most of the typical early Calculus II topics
- They're also very obviously applied and easy to motivate
- DE models allow us to simulate complicated systems that emerge from simple rules



Daniel V. Schroeder, `https://physics.weber.edu/schroeder/md/`

It's Different

- DEs can help to more immediately shake the students out of unhelpful high school habits
- They seem very obviously different than anything covered in Calculus I
- Studying them very obviously requires a different set of skills than what they might be used to from high school-level math



It’s Freeing

- The hand computations typically required in practice are often pretty simple
- Focusing more on applied DE models frees us to spend:
  - less time on artificially tricky toy problems
  - more time actually *doing* something with the results

Implementation

Differential Equations Unit

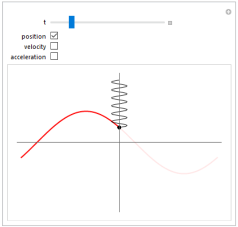
- The stated goals for the differential equations unit were:
  1. Create a differential equation to model a real-world system by describing its rates of change
  2. Determine the behavior of a system given its DE model by:
     a. solving the DE (exactly or numerically)
     b. qualitatively analyzing the DE (slope field, phase line, equilibrium analysis, etc.)

Differential Equations Unit

- The differential equations unit was deliberately designed to *avoid* explicitly reviewing Calculus I material for the first two weeks
- This was to prevent students from immediately getting into a memorization-based mindset

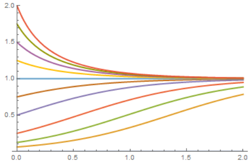
Differential Equations Unit

- The first week consisted of looking at differential equations models from various STEM applications
  - Exponential growth/decay, spring-mass, etc.
- The main goal was to illustrate that differential equations are interesting, important, and widely-used throughout STEM fields



Differential Equations Unit

- Fundamental concepts were discussed during Weeks 1–2
  - Defining a “solution” of a differential equation (this doubled as a covert review of differentiation rules)
  - Slope fields
  - Numerical solution methods
  - Equilibrium analysis
- By the end of Week 2 we could qualitatively analyze or numerically solve a DE, but not *exactly* solve one

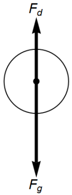


Differential Equations Unit

- The last major goal was “being able to exactly solve a differential equation”
- Week 3 was dedicated to reviewing the definite integral, the Fundamental Theorem of Calculus, and the Net Change Theorem
- All topics were contextualized in terms of how they apply to differential equations

Differential Equations Unit

- During Week 4 we covered the substitution method for integration
- This then allowed us to develop the method of separation of variables
- This concluded the Unit 1 material
  - Unit 1 exam
  - Unit 1 project (modeling freefall with drag)



The Rest of the Course

- The rest of the course was similarly organized into self-contained applications-first units
- Differential equations were reincorporated as a motivation for integration techniques and Taylor series

Changes for Next Time

Results

- Still too early to reach definite conclusions about the effects of these redesigns
- Overall student performance in class is similar to that of previous Fall sections
- The same has been true for all changes we've made to the course over the past several years

Results

- The redesign is certainly not *worse* than the usual version
- There's no reason *not* to use it
- Our instructors preferred the redesign and it's more fun to teach
- The potential benefits are good enough to stay with it

Student Perception

- Not much apparent student interest in applications
- A lot of students would prefer the class to just be a list of formulas and techniques to memorize

Changes in Framing

- Spend less time on the “business” of learning methods for hand computations
- Cut exact solution methods for differential equations and focus entirely on qualitative analysis and numerical solution
- Spend more time messing around with interesting, complex models for fun without needing to exactly solve or fully analyze them

Lean More Into Projects

- We did not have time for very many projects this Fall
- We plan to use them more regularly, and to incorporate group projects into the course

Conclusion

Your Input

- The changes we tried are ultimately fairly simple
- There are plenty of potential benefits to improve a student's first college-level math experience
- I would love for you to try this on your own and to hear about the improvements you make

Thank you!

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